Leapfrogging Cycles in International Competition

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June 2012

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Abstract

Technological leadership has shifted at various times from one country to another. We propose a mechanism that explains this perpetual cycle of technological leapfrogging in a two-country model including the dynamic optimization of an infinitely-lived consumer. In the model, each country accumulates knowledge stock over time because of domestic innovation and spillovers from foreign innovation. We show that if the international knowledge spillovers are reasonably efficient, technological leadership may shift first from one country to another, and then alternate between countries along an equilibrium path.

JEL Classification Numbers: E32; F44; O33

Keywords: Perpetual leapfrogging; innovation and growth cycles; endogenous innovation; knowledge spillovers

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*The author acknowledges the hospitality and support of Simon Fraser University, where some of the work in this paper was completed. He would like to thank the two anonymous referees and Timothy Kehoe (Co-Editor) for their helpful suggestions and valuable advice. He is also grateful to Taro Akiyama, Gadi Barlevy, Eric Bond, Kenneth Chan, Fumio Dei, Patrick Francois, Takamune Fujii, Sam Gamtessa, Tetsugen Haruyama, Jun-ichi Itaya, Ronald Jones, Kozo Kiyota, Takashi Komatsubara, Yoshimasa Komoriya, Jiang Li, Yan Ma, Sugata Margit, Junya Masuda, Tsukasa Matsuura, Hiroshi Mukunoki, Takeshi Ogawa, Takao Ohkawa, Masayuki Okawa, Rui Ota, Yuki Saito, Hitoshi Sato, Kenji Sato, Yasuhiro Takarada, Hirokazu Takizawa, Yoshihiro Tomaru, Eiichi Tomiura, Makoto Yano, Taiyo Yoshimi, Eden Yu, Ryuhei Wakasugi, and conference/seminar participants at the European Economic Association Annual Congress 2013, the Western Economic Association International Pacific Rim Conference 2013, the Canadian Economics Association Annual Conference 2014, Chukyo University, Chuo University, Kobe University, and Yokohama National University for their helpful comments and advice on earlier versions of the paper. The partial financial support of a Grant-in-Aid for Young Scientists (B) #23730198/26780126 is gratefully acknowledged.

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1 Introduction

Throughout history, technological leadership has shifted at various times from one country to another. For instance, during the early 17th century, Venice and Spanish Lombardy were among the technologically most advanced regions in Europe (Davids 2008, p. 2). Over the centuries, the "technological center of gravity of Europe then moved, residing at various times in Italy, southern Germany, the Netherlands, France, England, and then again in Germany" (Mokyr 1990, p. 207). Some economic historians even claim that the US had begun to lose its technological leadership as early as the early 1990s (Nelson and Wright 1992).

An important question is why such economic and technological leapfrogging takes place. An equally fundamental question is why technological leapfrogging has repeatedly occurred. The first question has been investigated in existing literature, in which technological leapfrogging is seen as been triggered by major exogenous changes in technology (Brezis, Krugman, and Tsiddon 1993).[1] In contrast, the cause of perpetual cycles in technological leapfrogging has scarcely been studied. While we may regard the perpetual cycles of leapfrogging as responses to the perpetual exogenous changes in technology, this explanation is essentially based on exogenous macro shocks in technology. The present paper offers an alternative explanation.

This paper develops a theory that explains the perpetual cycle in technological leadership as a market-driven equilibrium phenomenon that is free from exogenous shocks. For this purpose, we develop a new growth model that can capture in a tractable manner the process by which national technological leadership moves between countries along an equilibrium dynamic path. In doing so, we focus on endogenous innovation and international spillovers in a two-country setting with the dynamic optimization of consumption and saving by an infinitely-lived consumer. As the firms in a country develop innovations by investing resources, a knowledge stock accumulates in the home country, and this subsequently but only partially contributes to the accumulation of foreign knowledge because of international spillovers through foreign direct investment (FDI).[2]

By regarding technological leadership as the state whereby a given country develops the most innovations among all countries, we demonstrate that technological leadership by that country may shift to another country and then may alternate perpetually between countries. Specifically, we obtain two main results. (a) If the fundamental profitability of innovation is low, only the leading country innovates in equilibrium. In this case, leapfrogging never takes place. (b) If the fundamental profitability of innovation is sufficiently high, both leading and lagging countries engage in innovation. In this case, technological leadership can shift over time and will perpetually move back and forth between countries along an equilibrium path if international knowledge spillovers are reasonably efficient.

[1] See also Ohyama and Jones (1995), Motta, Thisse, and Cabrales (1997), Brezis and Tsiddon (1998), van de Klundert and Smulders (2001), and Desmet (2002). The present paper essentially differs from those analyses in its focus on perpetual cycles of leapfrogging, thus complementing these works by clarifying the intrinsically cyclical nature of national technological leadership.

[2] As argued by Brezis (1995), foreign capital plays a role in industrialization and development processes. We may also accept that international capital flows, as well as imports, are important channels for international knowledge spillovers, as discussed in the literature (Grossman and Helpman 1991; Feenstra 1996). See Branstetter (2006) for recent empirical evidence.

What we call the fundamental profitability of innovation is a composite of parameters that positively affects the equilibrium profit a firm earns from an innovation. This composite parameter completely determines into which case the world economy falls in equilibrium, namely, case (a), wherein only the leader innovates, or case (b), wherein both countries can innovate. We may think that we have experienced both regimes and transitions from one to the other. If we use the theory developed in the present paper to explain this, we may regard such regime transition as the result of an exogenous change in the parameters affecting the fundamental profitability of innovation. The implication is that a lagging country can leapfrog the leading country only when the fundamental profitability of innovation is high, which is when the lagging country can innovate in equilibrium.

The key driving force behind perpetual leapfrogging is the ability of a country to learn from foreign innovations. For example, a lagging country may learn much more from foreign innovations developed in a leading country than the leading country learns from those developed in the lagging country. Meanwhile, domestic innovations occur and build each country's knowledge stock. The analysis formally shows that leapfrogging is possible only when both countries innovate, where the lagging country has a dual engine of knowledge growth consisting of domestic innovation and foreign innovation diffused by spillovers. If a country can learn efficiently from diffused foreign innovations, technological leadership will perpetually alternate between countries. We can easily elaborate on why both countries innovate in equilibrium; the fundamental profitability of innovation can be sufficiently high, so that innovation pays even for the technologically lagging country. When the fundamental profitability is low, however, the lagging country does not innovate but simply receives spillovers from foreign innovation, resulting in a scenario where no leapfrogging occurs. This implies that the spillovers themselves can, at most, make the lagging country as innovative as, but *not more* innovative than, the leading country.

In addition to the leapfrogging cycles, leadership in (outward) FDI also shifts between countries since the technologically leading country is a foreign direct investor country in equilibrium in the model. Such a cycle in FDI, accompanied by leapfrogging cycles in technology, can be regarded as consistent with the fact that leadership in FDI has continually shifted in the real world. For example, the UK was the most active foreign direct investor country at the beginning of the 20th century, a role that shifted to the US by the middle of the century (Twomey 2000, p. 33, Table 3.2). Regarding an estimate for foreign investments including portfolios as a proxy of the amount of FDI (given that historical data on FDIs are not widely available), we can confirm that such foreign-investment leapfrogging took place by referring to Obsfeld and Taylor (2004, p. 52–3, Table 2.1). In addition, given that "the flow of capital from Holland to Great Britain, particularly in the second half of the eighteenth century, is well documented" (Brezis 1995), we can conjecture that the Netherlands would exhibit foreign-investment leadership before the UK. On the basis of these facts, one might think that leadership in FDI also fluctuates between countries. It is also worth noting that historically, the directions of technological leadership movement may seem similar to that of foreign-investment leadership movement (e.g., the UK to the US). This is consistent with our theoretical prediction.[3]

[3]A time lag can be seen in history, in which the technologically leading country is not necessarily the most active foreign direct investor country in the same period. A formal investigation of such a time lag is

The endogenous occurrence of perpetual leapfrogging is *not* new in the context of price competition between firms. For instance, the important paper by Giovannetti (2001) considers a duopoly in which firms considering infinite technological adoption set prices with Bertrand competition in the product market. Using this model, Giovannetti identifies the conditions whereby firms alternate in adopting the new technology, thereby representing a leapfrogging process. He shows that demand conditions, such as price elasticities, play a role in determining whether leapfrogging can be perpetual in Bertrand competition. Lee, Kim, and Lim (2011) have provided recent empirical support for this contention. In addition, some studies in the field of economic geography address both the theory of and empirical evidence for technological leapfrogging at the regional level (for example, Quah 1996a, b).[4] Different from the context of price competition, the present paper assumes that firms are monopolistically competitive as in the standard endogenous growth model (Romer 1990). Thus, there is no strategic interaction in the process of innovation, FDI, and pricing.[5]

The present study relates to the literature on innovation and growth cycles. In order to capture the cyclical growth phenomena in the simplest fashion, we follow Shleifer (1986), Deneckere and Judd (1992), Gale (1996), Francois and Shi (1999), and Matsuyama (1999, 2001) by assuming that patents last only for a single period in a discrete time model. This assumption implies that a single period is sufficiently long, which can be somewhere around 20 years. Given that in reality, many innovated consumption goods become obsolete before their patents expire, for the sake of simplicity, we assume that innovations become obsolete within a single period (which is fairly long). In line with these studies, which address neither leapfrogging nor its cycle, we assume the temporary nature of the monopoly enjoyed by innovators, which plays a role in explaining leapfrogging cycles in the growth process.[6]

In relation to this, in the present study, we view leapfrogging cycles as a discrete phenomenon.[7] This is in line with the literature on nonlinear equilibrium dynamics, in which a discrete-time growth model is commonly used for explaining complicated, real-world cycles (Nishimura and Yano 2008). Given that innovations often come in a cluster (Shleifer 1986), we believe that the discrete-time model can be a reasonable choice for explaining innovation-driven cycles such as leapfrogging in our model, although it is also essential to address this issue in a continuous-time setting as in Francois and Lloyd-Ellis (2003), who explain endogenous growth cycles in a continuous-time model of Schumpeterian growth.[8]

This study makes an important contribution to the theoretical literature by developing a new growth model with the dynamic optimization of an infinitely-lived consumer that can

left for future work.

[4] See Athreye and Godley (2009), Giovannetti (2013), and Petrakos, Rodríguez-Pose, and Rovolis (2005) for more recent research. In the political economy literature, Krusell and Ríos-Rull (1996) provide an endogenous explanation for a long cycle of stagnation and growth, similar to perpetual leapfrogging in the present paper, by focusing on vested interests in determining policies. See also Aghion, Harris, Howitt, and Vickers (2001) for perpetual leapfrogging at the firm level.

[5] See, for example, Hall (2008) and Harrington, Iskhakov, Rust, and Schjerning (2010) for research on dynamic strategic interaction in the competitive process.

[6] See also Iwaisako and Tanaka (2012) for endogenous cycles in a North–South product-cycle model with overlapping generations, in which innovation and imitation interact with each other to generate perpetual fluctuations in the world growth rate. However, leapfrogging does not exist in their model.

[7] See the discussion at the end of Section 3.3 on the use of a discrete-time model. See also footnote 14.

[8] See also Francois and Lloyd-Ellis (2008, 2009, 2013) for related studies.

explicitly capture how the (relative) national leadership in cutting-edge technology moves between countries over time along an equilibrium path. The beauty of the present model lies in its theoretical tractability and its ability to capture the main insights of the leapfrogging cycles in a simple setup, in which the equilibrium dynamical system is derived from the model as an autonomous one-dimensional system that allows us to track and explicitly illustrate an entire equilibrium path of national technological leadership between countries for any initial condition by means of a tractable phase diagram analysis. We achieve this by developing a new growth model that combines four standard elements: endogenous innovation, FDI, knowledge spillovers, and one-period patent length. In addition, the result is also novel to the existing literature on leapfrogging in demonstrating the intrinsically cyclical nature of national technological leadership; in our model, technological leadership perpetually fluctuates between countries on an equilibrium path. No research has addressed the equilibrium trajectory of national technological leadership or demonstrated the existence of perpetual cycles between countries.

2 Model

Time is discrete and extends from $-\infty$ to $+\infty$. Consider two countries, A and B, which have identical preferences and production and R&D technologies, differing only in their initial levels of innovation productivity. The countries are denoted by i or f ($i = A,\ B$; $f = A,\ B$), using a superscript for variables pertaining to the production side and a subscript for those pertaining to the consumption side.

There is a continuum of differentiated consumption goods in each period t. Each good is indexed by j. We follow the research and development (R&D)-based endogenous growth model with expanding variety (Romer 1990, Grossman and Helpman 1991) by assuming innovation as generating new varieties of goods. Given that we later allow for FDI, the country where a particular firm innovates and manufactures may change. Let $\Gamma^i(t)$ be the set of goods that are innovated in country i in period t, and let $\Lambda^i(t)$ be the set of goods manufactured in country i in period t.

2.1 Consumption

In each country, an infinitely lived representative consumer inelastically supplies L units of labor for production and R&D in every period. Note that the two countries are assumed to have equal labor forces, L. Each consumer is endowed with the same intertemporal utility function

$$U_i = \sum_{t=0}^{\infty} \beta^t \ln u_i(t),$$

where $\beta \in (0, 1)$ is the time preference rate. Temporary utility $u_i(t)$ is defined on the set $\{\Lambda^A(t) \cup \Lambda^B(t)\}$ of goods manufactured in both countries (free trade), taking the standard Dixit–Stiglitz form:

$$u_i(t) = \left(\int_{j \in \{\Lambda^A(t) \cup \Lambda^B(t)\}} x_i(j, t)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{1}$$

where $x_i(j,t)$ is the consumption of good j in country i. Parameter $\theta \in (0,1)$ denotes an inverse measure of the elasticity of substitution. Let $E_i(t) \equiv \int_{j\in\{\Lambda^A(t)\cup\Lambda^B(t)\}} p(j,t)x_i(j,t)dj$ be the spending in country i, where $p(j,t)$ denotes the price of good j. Solving the utility maximization problem in (1) leads to the demand function for good j, $x_i(j,t) = p(j,t)^{-(1/\theta)}E_i(t)/P(t)^{1-(1/\theta)}$, where $P(t)$ is the price index.[9] Aggregating these expressions, we obtain the derived aggregate demand, $x_A(j,t) + x_B(j,t) \equiv x(j,t)$, as

$$x(j,t) = \frac{E(t)p(j,t)^{-(1/\theta)}}{P(t)^{1-(1/\theta)}}, \tag{2}$$

where $E(t) = E_A(t) + E_B(t)$ is the aggregate spending in period t. The price elasticity of demand is constant at θ^{-1} for any j.

Solving the dynamic optimization of the consumer's utility for consumption and saving decisions under the intertemporal budget constraint results in the usual Euler equation $E_i(t+1)/E_i(t) = \beta(1+r(t))$, where $r(t)$ is the interest rate in period t. We obtain

$$\frac{E(t+1)}{E(t)} = \beta(1+r(t)). \tag{3}$$

2.2 Innovation, FDI, and manufacture

A single firm innovates and monopolistically supplies each differentiated consumption good, following the standard endogenous growth framework (Romer 1990).[10] Innovating a new good takes one period. In each period, say $t-1$, a firm in country i can innovate one technology to produce a new differentiated good at the end of the period, $t-1$, by investing $1/K^i(t-1) \equiv k^i(t-1)$ units of domestic labor in R&D activity.[11] Here $K^i(t-1)$ denotes the technology level in innovation for country i in period $t-1$, and innovation is achieved entirely via domestic labor resources (no R&D outsourcing). In the subsequent period t, the firm will set up a production plant. In doing this, the firm can choose the country in which to manufacture the good in order to maximize monopolistic profits. In equilibrium, as foreign profits may be greater, the firm may transfer production to a foreign country through FDI. This is the channel for innovation diffusion in our model.[12]

[9] As is well known, the index is defined as $P(t) = \left(\int_{j\in\{\Lambda^A(t)\cup\Lambda^B(t)\}} p(j,t)^{1-(1/\theta)}dj\right)^{\frac{1}{1-(1/\theta)}}$.

[10] As Romer (1990) explains, this simplified setting is only a convenience since "(w)hether the owner of the patent manufactures the good itself or licenses others to do so, it can extract the same monopoly profit." One potential oversimplifying factor here is the lack of explicit and costly adoption of innovation, which should be a limiting factor for the analysis in a broader context. While a complete analysis on costly innovation adoption is beyond the scope of this paper, we can incorporate a process of costly adoption of innovation without essentially changing the results by using a very simple setting; see Appendix C (not for publication).

[11] Following the literature (Romer 1990, Matsuyama 1999), we consider a deterministic innovation process for the sake of simplicity, although without any qualitative change in our results, we can consider a simple stochastic innovation process in which success probability for a firm to innovate a new good is endogenous and increases with the firm's R&D investment (see Appendix C (not for publication)). However, if we assume that the time for each innovation to be completed was not fixed at one period but was stochastic, the analysis becomes intractable. We leave the question of how such stochastic timing of innovation impacts leapfrogging for future research.

[12] In line with the literature on international trade and growth (Lai 1998), we do not distinguish between the various forms of production transfer, including fully and partly owned subsidiaries and licensing.

We assume a simple production technology. There are constant returns to scale in the production of any good j and the productivity of labor is the same in both countries, which is normalized to be one.[13] The marginal cost in country i is thus equal to the wage rate in country i, $w^i(t)$. When the firm chooses to manufacture in country i in period t, captured by $j \in \Lambda^i(t)$, it produces $x(j,t)$ units of good j by using labor in country i. The standard profit maximization problem is written as

$$\max_{(p(j,t),\ x(j,t))} \pi(j,t) = p(j,t)x(j,t) - w^i(t)x(j,t)$$

subject to the market demand function (2). Since, by (2), the price elasticity of each good j is constant at $1/\theta$, the firm sets a monopolistic price of $p(j,t) = w^i(t)/(1-\theta) \equiv p^i(t)$. By substituting this into (2), we obtain the demand and profit functions as

$$x(j,t) = \frac{E(t)p^i(t)^{-(1/\theta)}}{P(t)^{1-(1/\theta)}} \equiv x^i(t) \tag{4}$$

and

$$\pi(j,t) = \theta E(t)\left(\frac{p^i(t)}{P(t)}\right)^{1-(1/\theta)} \equiv \pi^i(t) \tag{5}$$

for $j \in \Lambda^i(t)$ ($i = A,\ B$). As firms prefer the country where profits are higher, the discounted present value of the firm innovating in country i in period $t-1$ is expressed as

$$V^i(t-1) = \frac{\max\{\pi^A(t), \pi^B(t)\}}{1+r(t-1)} - w^i(t-1)\ k^i(t-1). \tag{6}$$

In order to capture cyclical phenomena in the simplest fashion, we follow Shleifer (1986), Deneckere and Judd (1992), Gale (1996), Francois and Shi (1999) and Matsuyama (1999, 2001) by assuming that patents last only for one period.[14] This assumption implies that the length of a unit period is sufficiently long, which can be around 20 years, in keeping with the duration of real-world patents. Given that in reality, many innovated consumption goods become obsolete before their patents expire, we may assume that innovations are made obsolete within a single period (which in our model is fairly long).[15] As shown below, this

[13] Here we simply consider that efficiency in manufacturing normalizes across countries. We can extend this simple setting by allowing for country-specific manufacturing efficiency and endogenous technological progress. In such an extended model, we can easily verify that the comparative advantage between R&D and manufacturing (rather than the absolute advantage in R&D) plays an important role in perpetual leapfrogging, although there is no fundamental change in the results and their implications for perpetual leapfrogging.

[14] This assumption implies that all patents start and expire at the same time, although in reality patents overlap. We may deal with this undesirable property by interpreting the length of a period as very long (e.g., 40 years) and dividing each period into subperiods (e.g., two 20-year periods), although we need a continuous-time model to completely fix this problem. In the present paper, we view leapfrogging as a discrete-time phenomenon and leave this issue for future work.

[15] This assumption may also be justified if each innovation is interpreted as fairly specific. For example, "innovation" in this model would be represented by the specific innovation associated with iPhone 4S or smartphones instead of cell phones or information technology more generally.

assumption makes the analysis tractable without any fundamental change in the results.[16] Finally, free entry guarantees that the net value of a firm is not positive in equilibrium: $V^i(t-1) \leq 0$ for each i.

2.3 Knowledge accumulation and spillovers

Technology in innovation $K^i(t)$ advances with knowledge accumulation. Following Romer (1990), we assume intertemporal knowledge spillovers in innovation: current innovations contribute to the accumulation of the stock of knowledge $K^i(t)$, with which the cost of innovation, $k^i(t) = 1/K^i(t)$, reduces over time. Here, as is standard, the technology level in innovation $K^i(t)$ is interpreted as the knowledge stock in innovation.

The knowledge stock of a country consists of cumulative innovations of two types: home and foreign innovations. Define

$$N^i(t) \equiv \int_{\Gamma^i(t)} dj \text{ and } M^i(t) \equiv \int_{j \in \Gamma^f(t-1) \cap \Lambda^i(t)} dj. \tag{7}$$

Here, $N^i(t)$ denotes the number of innovations developed in country i in period t and $M^i(t)$ denotes the number of products that are innovated in period $t-1$ in country f $(f \neq i)$ and then flow into country i from country f in period t. Following Romer (1990), we assume that the knowledge stock $K^i(t)$ linearly depends on the sum of domestic innovations that are developed up to the beginning of period t; i.e., $N^i(t-1) + N^i(t-2) + \cdots$, where $N^i(s)$ is a familiar proxy for the flow of knowledge generated as a by-product of the innovations achieved in period s. We also assume that the international knowledge spillovers as an externality accompany FDI, such that each country learns from its foreign innovation inflows. Hence the knowledge stock of country i also depends on the sum, $M^i(t-1)+M^i(t-2)+\cdots$. Accordingly, we describe the knowledge stock using

$$K^i(t) = \sum_{s=-\infty}^{t} \big(N^i(s-1) + \mu M^i(s-1)\big) \text{ with } \mu \leq 1, \tag{8}$$

where the parameter $\mu \in [0, 1]$ captures the efficiency of the contribution of international knowledge spillovers through foreign innovation inflows to knowledge accumulation and thus technological progress occurs. The efficiency of international knowledge spillovers increases with μ. If $\mu = 1$, spillovers are as efficient as domestic spillovers; if $\mu = 0$, there is no learning at all from foreign innovations. For the sake of explanation, we rewrite (8) as a flow as follows

$$K^i(t+1) - K^i(t) = N^i(t) + \mu M^i(t). \tag{9}$$

Considering (9), one may conjecture that spillovers $M^i(t)$ by themselves can cause a reversal of $K^i(t) > K^f(t)$: $K^i(t+1) < K^f(t+1)$ might hold by taking a sufficiently large $M^f(t)$. Leapfrogging may be able to occur simply through spillover $M^f(t)$ from country i to f. However, this conjecture is not the case with the present model because the example (of such

[16] Note that, in the next subsection, we assume that obsolete innovations stay "alive" in the sense that they continue to contribute to the current knowledge stock, although they are not explicitly traded in the marketplace. Whether they are traded or not is not important for our main story explained later.

a large $M^f(t)$) is not consistent with (8) and (9) for the following reason. Equation (8) says that innovation in country i, $N^i(t-1)$, not only contributes to the foreign knowledge, $K^f(t+1)$, through spillovers of $M^f(t)(=N^i(t-1))$ but also increases the domestic knowledge $K^i(t)$ in the previous period. Therefore, it is not possible to arbitrarily take a large $M^f(t)$ with $K^i(t)$ constant. *As $M^f(t)$ becomes large, by (8), $K^i(t)$ and $K^i(t+1)$ must also become large at a higher rate than, or at least the same rate as, $K^f(t+1)$ does.* We cannot artificially make $K^i(t+1) < K^f(t+1)$ by controlling $M^f(t)$ only. As we will see later, a sufficient number of domestic innovations, $N^f(t)$, is essential for the reversal of $K^i(t) > K^f(t)$ (i.e., leapfrogging). In summary, so long as we choose an identical equilibrium path, spillovers $M^f(t)$ by themselves cannot cause a reversal of $K^i(t) > K^f(t)$.

It is also worth pointing out that in (8) and (9), we assume that knowledge develops horizontally, rather than vertically. That is, we assume that knowledge accumulates as innovations are added to old innovations, not as innovations replace old innovations. Under this horizontal modeling of knowledge accumulation, a country's knowledge stock can be related to a collection of "blueprints" for the country, i.e., how many goods the country knows how to produce. In addition, different countries innovate along different lines. This captures the fact that technologies or products made in different countries are sometimes at least slightly differentiated. Thus, the knowledge stock of country i can accumulate as foreign innovations ($M^i(t)$) simply add to, rather than replace, domestic innovations ($N^i(t)$). Although the results would become richer if the model also included the replacement of technologies or a knowledge stock as a vertical ladder, in this study, we focus on the above-mentioned horizontal aspect of knowledge with (8) and (9), which can help us highlight our main point.[17]

3 Technological Leadership in Equilibrium Dynamics

In this section, we prove the main result that technological leadership may endogenously fluctuate over time, thereby perpetually moving back and forth between countries along an equilibrium path. Before proceeding, we provide a formal definition of the concept of technological leadership. Taking into account the notion in economic history (Davids 2008),[18] we refer to a country that develops the most innovations among the countries as the technological leader, and to a country that develops few innovations as a lagging country. In the present model, and as will be made apparent later, this definition implies leadership as the state whereby a given country has the highest innovation productivity among the countries, which is consistent with the definition in existing literature. Thus, in equilibrium, country

[17] One method for vertically capturing knowledge accumulation *within the present setting* is to consider obsolescence of knowledge by assuming that knowledge accumulates as innovations *partially* replace old innovations, rather than simply being added to old innovations as in (8). For example, we can introduce a rate of knowledge destruction, say $\delta \in [0,1]$, into (8), with which (9) would be revised to $K^i(t+1) - K^i(t) = N^i(t) + \mu M^i(t) - \delta K^i(t)$. As long as the two countries have identical δ, we can demonstrate that our main result is robust to this extension. Otherwise, it would be possible to show a result similar to that of the present paper in a quality-ladder variant of the present model; see Appendix D (not for publication) for a formal explanation.

[18] Davids (2008) considered that a country that has technological leadership plays an initiating role in the development of new technologies across a wide variety of fields.

i innovates more if and only if its innovation productivity is higher; $N^i(t) > N^f(t)$ if and only if $K^i(t) > K^f(t)$. For simplicity, we use $K^i(t) > K^f(t)$ to designate country i as the technological leader, and we refer to any reversal of the leading position as technological leapfrogging.

Without loss of generality, we assume that country A is the leading country in period t, $K^A(t) > K^B(t)$ (and thus $N^A(t) > N^B(t)$ to be shown in equilibrium), and we refer to this situation as regime A. If $K^A(t) < K^B(t)$ (and thus $N^A(t) < N^B(t)$ to be shown in equilibrium), we refer to it as regime B.

In any period, this model can be regarded as a variant of a conventional two-good Ricardian model, where the two outputs considered are innovation and production. Given $K^A(t) > K^B(t)$, there are potentially three possible specialization patterns in period t:[19] (1) one in which both countries engage in manufacturing and only one country engages in R&D, (2) one in which both countries engage in R&D and only one country engages in manufacturing, and (3) one in which both countries are specialized. It is useful to define a new variable, $N(t) = \int_{j\in\{\Lambda^A(t)\cup\Lambda^B(t)\}} dj$, which is the total number of goods manufactured in t, satisfying $N(t) = N^A(t-1) + N^B(t-1)$.

Let us explain the essential role of FDI in how the economy endogenously falls into each pattern *in the pre-equilibrium process*, that is, the process through which the economy reaches equilibrium. Suppose that a wage differential between the countries exists on an off-equilibrium path, say $w^A(t) > w^B(t)$. In lagging country B, wages are lower and thus *profits are higher*. This gives rise to an arbitrage opportunity; firms in leading country A sequentially engage in FDI and move to lagging country B for higher profits. This gradually increases the labor demand by manufacturing in lagging country B, which in turn generates a continual increase in the wage rate in lagging country B, $w^B(t)$.

Three possibilities exist. (1) One possibility is that $w^B(t)$ continues to increase until it is equal to the wage rate in leading country A, $w^A(t)$, in which case wages are internationally equated in equilibrium $w^A(t) = w^B(t)$, whereupon some firms stay and produce goods in leading country A, while the other firms shift their production to lagging country B. This situation corresponds to pattern (1), in which both countries engage in manufacturing and R&D investment is always unprofitable for lagging country B because the technological gap $K^A(t) > K^B(t)$ with $w^A(t) = w^B(t)$ generates an R&D cost gap $w^A(t)/K^A(t) < w^B(t)/K^B(t)$. (2) Another possibility is that the wage differential remains when all firms move to lagging country B. In this case, leading country A's wage rate is higher even in equilibrium, $w^A(t) > w^B(t)$. Lagging country B uses this cost advantage to engage in R&D if the technology gap $K^A(t) > K^B(t)$ is not too large, in which case both countries engage in R&D. This represents the specialization pattern (2).[20] Finally, in case (3), if the technology gap $K^A(t) > K^B(t)$ is sufficient, the net benefit of innovation cannot be positive for lagging country B despite it having the cost advantage, $w^A(t) > w^B(t)$. This corresponds to the last possibility relating to pattern (3). In what follows, we formally confirm those processes by deriving the necessary and sufficient conditions for the pattern into

[19] The model contains no zero-innovation equilibrium because the so-called Inada property is assumed in the constant elasticity of substitution utility function (1), which is standard in the literature.

[20] Specialization pattern (2) implies that in equilibrium, the technology gap $(K^A(t)/K^B(t))$ is exactly equal to the wage ratio $(w^A(t)/w^B(t))$; otherwise, the countries would not be both engaging in R&D.

which the economy falls in equilibrium.

3.1 No Leapfrogging

We investigate the specialization pattern (1) where both countries produce goods in equilibrium. First, we will derive a necessary and sufficient condition under which both countries manufacture goods in equilibrium. Then, we will demonstrate that leapfrogging can never take place in this equilibrium.

When both countries produce goods in equilibrium, as explained above, the wage rates must be equated in equilibrium since any manufacturing firm would go to either country through FDI if there remains a wage differential. Thus, $w^A(t) = w^B(t) = w(t)$ must hold in equilibrium for the pattern (1), implying $p^i(t) = p(t)$ and thus $x^i(t) = x(t)$ by (4). Then, due the technology gap $K^A(t) > K^B(t)$, the cost for innovation is always lower in leading country A, i.e., $w(t)/K^A(t) < w(t)/K^B(t)$. As a result, only leading country A innovates; $\Gamma^A(t) \neq \emptyset$ and $\Gamma^B(t) = \emptyset$. By (7), we have $N^A(t) > 0$ and $N^B(t) = 0$. By $N(t+1) = N^A(t) + N^B(t)$, we also have $N(t+1) = N^A(t)$. As this situation is similar to the North–South product-cycle model à la Krugman (1979) and Helpman (1993) where only the North innovates and both the North and the South manufacture, we may refer to this pattern as a "North–South regime."[21]

Because leading country A innovates in equilibrium, the free-entry condition requires $V^A(t) = 0$ in equilibrium. By incorporating $w^A(t) = w^B(t) = w(t)$ into (6), it also requires $V^B(t) < 0$ in equilibrium. Since $p^i(t) = p(t)$, using (5), the discounted present value of an innovation in country i in (6) can be expressed as

$$V^i(t) = \frac{1}{1+r(t)} \frac{\theta E(t+1)}{N(t+1)} - w^i(t)\ k^i(t). \tag{10}$$

Substituting into (10) the Euler equation $1+r(t) = E(t+1)/\left(\beta E(t)\right)$ from (3), the free-entry condition $V^A(t) = 0 > V^B(t)$ becomes

$$\frac{\beta\theta E(t)}{N(t+1)} = w(t)\ k^A(t) < w(t)\ k^B(t). \tag{11}$$

The first equality in (11) ensures that the discounted value of an innovation $(\beta\theta E(t)/N^A(t))$ and the cost $(w(t)\ k^A(t))$ are balanced in leading country A. The second inequality in (11) simply means that the cost of an innovation is lower in the leading country $(w(t)k^A(t))$ than in the lagging country B $(w(t)k^B(t))$.

The labor market-clearing conditions for country i is given by

$$L = \int_{\Gamma^i(t)} k^i(t)dj + \int_{\Lambda^i(t)} x^i(t)dj. \tag{12}$$

[21] Here, we assume that the North is a country that innovates; however, if the North (the South) was defined as a country where the wage rate is higher (lower) as is also usual in the literature, these North/South labels could be misleading. Nevertheless, we use these labels because we can easily control the international wage differential in the present model by incorporating into the model an international differential in manufacturing productivity.

The quantity of a good, $x^i(t)$, can be derived from (4) and (7) as $x^i(t) = (1-\theta)E(t)/\left(w(t)N(t)\right)$ for each i. Together with this, by eliminating the country index i from (12),[22] we can obtain the world labor constraint as

$$2L = N^A(t)\ k^A(t) + (1-\theta)\frac{E(t)}{w(t)}, \tag{13}$$

in which $\Gamma^B(t) = \emptyset$ is used. The left-hand side in (13) is the world supply of labor, and the right-hand side is the world demand for labor from both the innovation sector in leading country A and the manufacturing sectors in both countries.

In order to determine the equilibrium flow of innovation made in period t, we will eliminate the term $E(t)/w(t)$ from the world labor market-clearing condition (13), using the free-entry condition (11). Then, noting $k^A(t) = 1/K^A(t)$ and $N(t+1) = N^A(t)$, the flow of innovation in period t is derived as

$$N^A(t) = K^A(t)\frac{2L\Theta}{1+\Theta} \quad \text{and } N^B(t) = 0, \tag{14}$$

where Θ is a composite of the parameters β and θ. The formal definition of Θ is given by

$$\Theta \equiv \frac{\beta\theta}{1-\theta}. \tag{15}$$

The parameter Θ captures the discounted present value of a markup ratio (i.e., the ratio of price to marginal cost) for the firm,[23] which increases with time preference β and decreases with the elasticity of substitution θ^{-1}.[24] Since parameter Θ positively affects equilibrium profits from innovating a good, we refer to it as an innovation's fundamental profitability. Note that the equilibrium profit itself is endogenously determined with other endogenous variables, and it changes over time, although fundamental profitability Θ is exogenously given by (15). Equation (14) shows that the innovation flow $N^A(t)$ increases with the knowledge stock $K^A(t)$ and an innovation's profitability.

Now, we can derive the number of goods that are manufactured in each country. Noting (12) with $\Gamma^B(t) = 0$, we have $L = \int_{\Lambda^B(t)} x^B(t)dj$. Then, by (11) and (14), we obtain

$$\left(\int_{\Lambda^A(t)} dj\right) = \frac{1-\Theta}{2}N(t) \quad \text{and} \quad \left(\int_{\Lambda^B(t)} dj\right) = \frac{1+\Theta}{2}N(t). \tag{16}$$

To ensure that leading country A manufactures goods in equilibrium, i.e., $\int_{\Lambda^A(t)} dj > 0$, it must hold that

$$\Theta < 1. \tag{17}$$

The parameter condition (17) is more likely to hold when the time preference β is smaller and the price elasticity of substitution θ^{-1} is higher. The condition (17) can be shown as necessary and sufficient for an economy to fall within the North–South regime.

[22] We do this by summing both sides of (12) over i.

[23] Note that the constant elasticity of substitution θ^{-1} is equal to the price elasticity of each good j, which determines the markup ratio as $1/(1-\theta)$.

[24] Li (2001) argues that the evidence regarding whether there is any conventional value or a range of values for the elasticity of substitution is inconclusive. For example, Broda and Weinstein (2006) show that the elasticity of substitution is, on average, greater than two, but tends to decline over time and is actually less than two in some sectors (e.g., motor vehicles).

Lemma 1 *The economy falls in the North–South regime if and only if* $\Theta < 1$, *in which both countries produce goods and only the leading country innovates.*

Proof. See Appendix A. ■

Why do both countries manufacture for $\Theta < 1$? In other words, why are the equilibrium wages internationally equated for $\Theta < 1$? Recall the essential role of FDI explained at the beginning of this section. An international wage differential such as $w^A(t) > w^B(t)$ could exist, because the only source of an international wage differential in the model is leading country A's advantage in R&D productivity, $K^A(t) > K^B(t)$. As long as $w^A(t) > w^B(t)$, a firm in leading country A engages in FDI to shift their production to lower-wage country B, which tightens labor resource scarcity in country B. As a result, the wage rate in lagging country B, $w^B(t)$, gradually rises as the economy reaches equilibrium. Lemma 1 implies that when the fundamental profitability of innovation Θ is small (i.e., $\Theta < 1$), the potential for an international wage gap is also small,[25] and international wages can become equated at the point in time at which *some* (not all) firms in leading country A move to lagging country B through FDI. Ultimately, both countries manufacture, and only leading country A innovates, as $w^A(t) = w^B(t)$ naturally creates an R&D cost advantage for leader A, such as $w^A(t)/K^A(t) < w^B(t)/K^B(t)$.

Thus far, we have three important conditions. Inequality (17) is the necessary and sufficient condition for an economy to fall within the North–South regime in equilibrium, under which both countries produce goods and only the leading country innovates in equilibrium. Equations (14) and (16) determine the innovation flow and the fractions of manufactured goods, respectively, in the North–South regime.

In what follows, we demonstrate that in the North–South regime, leapfrogging never occurs even if spillovers are completely efficient ($\mu = 1$). By (14) and (9), the growth of knowledge can be expressed as follows:

$$K^A(t+1) = \left(\frac{2L\Theta}{1+\Theta} + 1\right) K^A(t) \tag{18}$$

and

$$K^B(t+1) = \mu M^B(t) + K^B(t), \tag{19}$$

where $M^B(t) = \int_{\Lambda^B(t)} dj = (1+\Theta)\,N(t)/2$ by (16).

As $K^A(t)$ is given by history, (18) fully determines the growth of knowledge in leading country A. Apparently, (19) does not determine $K^B(t+1)$ without additional historical assumptions because the amount of spillovers $M^B(t) = (1+\Theta)\,N(t)/2$ depends on the number of goods $N(t) = N^A(t-1) + N^B(t-1)$, which is determined by innovation activities undertaken in the previous period, $t-1$. Nevertheless, as shown in our first theorem, regardless of past innovation activities, $N^i(s)$ for $s \leq t-1$, leapfrogging never occurs in the North–South regime.

[25] It is important to elaborate why the potential for an international wage differential is larger when Θ is larger. Given that the countries are identical except for $K^i(t)$, the productivity gap in R&D ($K^A(t) > K^B(t)$) is the only source for an international wage differential in our model. Thus, the potential for a wage differential is is enhanced by a larger share of R&D investment, which naturally increases with the fundamental profitability of innovation Θ. The larger the value of Θ, the larger the potential for an international wage differential.

Theorem 1 (No leapfrogging with lower profitability) *Suppose that the fundamental profitability of an innovation* Θ *falls below* 1*. Then, under the infinitely lived agent's dynamic optimization, only the leading country innovates in equilibrium (the North-South regime). In this case, leapfrogging never takes place.*

Proof. By Lemma 1, the parameter restriction $\Theta < 1$ ensures that only the leading country innovates. By (16), $M^B(t) = (1+\Theta)\,N(t)/2$. (a) Assume $K^A(t-1) > K^B(t-1)$. By the expression of $N^A(t-1)$ in (14), with $K^A(t-1) = N^A(t-1) + K^A(t)$ from (9), substituting $N(t) = N^A(t-1)$ into (19) derives

$$K^B(t+1) = K^A(t)\frac{\mu L\Theta\,(1+\Theta)}{(2L+1)\,\Theta+1} + K^B(t). \tag{20}$$

From (18) and (20), we can show that $K^A(t+1) > K^B(t+1)$ holds so long as $K^A(t) > K^B(t)$, noting $\Theta < 1$ and $\mu < 1$. (b) Assume $K^A(t-1) < K^B(t-1)$. By symmetry, noting $N(t) = N^B(t-1)$, the analogous procedures derive

$$K^B(t+1) = \left(\frac{\mu L\Theta\,(1+\Theta)}{(2L+1)\,\Theta+1} + 1\right) K^B(t). \tag{21}$$

From (18) and (21), $K^A(t+1) > K^B(t+1)$ holds so long as $K^A(t) > K^B(t)$, given $\Theta < 1$ and $\mu < 1$. This proves that $K^A(t) > K^B(t)$ cannot be reversed for the subsequent period when $\Theta < 1$, regardless of whether either country was a leader in the previous period $t-1$. ■

We now elaborate upon the theories of why economies with lower fundamental profitability of innovation cannot experience leapfrogging. Leapfrogging can stem from two sources: knowledge growth from domestic innovation and spillovers from foreign innovation. In this regime, however, the lagging country (country B) only receives spillovers from $M^B(t)$, the foreign innovations developed in the leading country (country A). No domestic innovations are produced in the lagging country. Given that the leading country also gains from innovations $M^B(t)$ (which are included in $K^A(t)$) even more efficiently than, or at least as efficiently as, the lagging country, *the spillovers alone can only make the lagging country as innovative as, but not more innovative than, the leading country.* Thus, leapfrogging never occurs. As shown later, leapfrogging is possible only if the lagging country not only receives spillovers from the leading country but also innovates by itself.

Why does only the leading country innovate when the fundamental profitability is low such that $\Theta < 1$? We have already answered this question formally in this section; thus, it now suffices to provide an intuitive explanation. Since $\Theta = \beta\theta/(1-\theta)$, lower Θ is associated with lower β and higher θ^{-1}. A lower time preference β results in a higher interest rate $r(t)$, which decreases the discounted value of profit $\pi(t)$. A higher elasticity of substitution θ^{-1} implies a lower markup ratio $(1/(1-\theta))$ and a lower profit $\pi(t)$. The inequality condition $\Theta < 1$ intuitively requires an innovation's discounted value to be fairly low. That is, the value is too low for lagging country B to innovate by itself. In other words, where the fundamental profitability of an innovation Θ (depending on time preference rate β and elasticity of substitution θ^{-1}) is higher, the discounted benefit from an innovation would be higher and thus innovation would be profitable, even for firms in the lagging country. Finally, we may summarize this by stating that when an innovation has low fundamental profitability, leapfrogging does not take place because the lagging country does not innovate.

3.2 An illustration

To further illustrate the international dynamics of knowledge in the North–South regime, we assume that leading country A has retained leadership in the past; i.e., $N^A(s) > 0 = N^B(s)$ and thus $K^A(s) > K^B(s)$ for $s = t,\ t-1,\ \cdots$. This consideration is reasonable given that Theorem 1 shows that leapfrogging never takes place. The growth of knowledge follows (18) and (20) for any $s \geq t$. Define $\psi(t) = K^A(t)/(K^A(t) + K^B(t))$, which stands for the knowledge ratio for country A. We can derive the dynamic system for $\psi(t)$ as follows. Noting $K^A(t) > K^B(t)$,

$$\psi(t+1) = \frac{(a_1 + 1)\,\psi(t)}{1 + (a_1 + \mu a_2)\,\psi(t)} \text{ for } \psi(t) \in (0.5, 1), \tag{22}$$

where a_1 and a_2 are positive numbers determined by β, θ, and L.[26] By applying the above procedures to the case of $\psi(t) \in (0, 0.5)$ where country B is the leading country, we can easily derive the following dynamic system:

$$\psi(t+1) = \frac{(1 - \mu a_2)\,\psi(t) + \mu a_2}{1 + (1 - \psi(t))\,(a_1 + \mu a_2)} \text{ for } \psi(t) \in (0, 0.5). \tag{23}$$

Note that $a_1 < 1$ and $a_2 < 1$ if $\Theta < 1$. We thus can verify that so long as $\Theta < 1$, the steady state is unique and higher than 0.5 for (22) and lower than 0.5 for (23).

Figure 1 illustrates the phase diagram for systems (22) and (23) with their steady states, ψ_A^* and ψ_B^*. As shown, any path starting in the situation where country A (B) is the leading country stably converges to a steady state; $\psi(s) > (<) 0.5$ for all $s > t$ if $\psi(t) > (<) 0.5$. Thus, this phase diagram shows that no leapfrogging occurs in the case where the fundamental profitability of an innovation is lower.

3.3 Leapfrogging cycles

We now consider situations where only the lagging country manufactures (specialization patterns (2) and (3)). We demonstrate that these patterns are realized in equilibrium if and only if the parameters satisfy $\Theta > 1$. Then, we show that in this case, leapfrogging can take place in equilibrium.

By Lemma 1, it is implied that only one country manufactures in equilibrium if and only if $\Theta > 1$. We can easily exclude the case where only leading country A manufactures.[27] Therefore, in equilibrium, only the lagging country manufactures if and only if $\Theta > 1$, where a wage differential such as $w^A(t) \geq w^B(t)$ must exist as manufacturing firms in lagging country B would go to leading country A if $w^A(t) < w^B(t)$; thus, $\pi^A(t) < \pi^B(t)$ held. Because this gives the lagging country a cost advantage, there are two possibilities: one where both countries innovate (pattern (2)) and another where only leading country A innovates (pattern (3)).

[26] The formal definitions are:

$$a_1 \equiv \frac{2L\Theta}{1+\Theta} \text{ and } a_2 \equiv \frac{L\Theta(1+\Theta)}{1+\Theta(2L+1)}.$$

[27] Only leading country A produces goods if and only if $w^A(t) < w^B(t)$. This implies $w^A(t)/K^A(t) < w^B(t)/K^B(t)$ with $K^A(t) > K^B(t)$ in which only leading country A innovates. No labor demand exists in lagging country B, which is inconsistent with the market clearing condition.

We first investigate pattern (3), where only leading country A innovates and only lagging country B produces goods. For the sake of explanation, we may refer to this as a full North–South regime. Since innovation takes place only in leading country A in equilibrium, the free entry condition must hold as $0 = V^A(t) > V^B(t)$. Since manufacturing takes place only in lagging country B in equilibrium, a wage differential must remain as $w^A(t) < w^B(t)$. Using these conditions (see the proof of Lemma 1 in Appendix A for details), the wage rate in the lagging country is derived by its labor market-clearing condition as $w^B(t) = (1-\theta) E(t)/L$, the wage rate in the leading country by the free entry condition as $w^A(t) = \beta\theta K^A(t)E(t)/N^A(t)$, the innovation flow in leading country A as $N^A(t) = LK^A(t)$, and the innovation flow as $N^B(t) = 0$. On the one hand, the free entry condition $V^A(t) > V^B(t)$ implies $w^A(t)/K^A(t) < w^B(t)/K^B(t)$ in equilibrium, which can be rewritten as $K^A(t)/K^B(t) > \Theta$. On the other hand, the wage differential condition $w^A(t) > w^B(t)$ can be rewritten as $\Theta > 1$. Combining these two yields

$$\frac{K^A(t)}{K^B(t)} > \Theta > 1. \tag{24}$$

Lemma 2 *The economy falls in the full North–South regime if and only if $K^A(t)/K^B(t) > \Theta > 1$, in which only the leading country innovates and only the lagging country produces goods.*

Proof. See Appendix A. ■

Lemma 2 implies that so long as $\Theta > 1$, innovation is more profitable and the potential for an international wage differential is greater (see footnote 25); the wage differential $w^A(t) > w^B(t)$ must remain in equilibrium, in contrast with that in the North–South case with $\Theta < 1$.[28] By using this wage differential $w^A(t) > w^B(t)$, lagging country B can enter the innovation market; however, this is not the case as long as leading country A's technological advantage $K^A(t)/K^B(t)$ is larger than Θ, so that the R&D cost is still lower in leading country A, i.e., $w^A(t)/K^A(t) < w^B(t)/K^B(t)$.

Incorporating $N^A(t) = LK^A(t)$ and $N^B(t) = 0$ into (9), knowledge accumulation in the full North–South regime (where country A is assumed to be a leading country) is as follows:

$$K^A(t+1) = (L+1)\, K^A(t) \text{ and } K^B(t+1) = K^B(t) + \mu L K^A(t). \tag{25}$$

Using (24), we can demonstrate that the full North–South regime is unstable in the following sense.

Theorem 2 *Suppose that the profitability of an innovation Θ exceeds 1 and that the international technological gap $K^A(t)/K^B(t)$ is larger than Θ. Then, under dynamic optimization by the infinitely lived agent, only the leading country innovates and only the lagging country manufactures (the full North–South regime). In this case, leapfrogging never takes place in equilibrium. Moreover, any equilibrium path of $K^A(t)/K^B(t)$ starting from this regime converges to the steady state within the regime if $\mu < 1/\Theta$ and eventually moves beyond this regime (i.e., $K^A(t)/K^B(t)$ eventually becomes below Θ) if $\mu > 1/\Theta$.*

[28] The answer to the question of why the equilibrium wage differential between countries exists for $\Theta < 1$ is similar and symmetric to that of the question why of the wages are equated for $\Theta < 1$. See the discussion after Lemma 1.

Proof. By (24), the equilibrium system can be derived as

$$\psi(t+1)=\frac{(L+1)\,\psi(t)}{1+(1+\mu)\,L\psi(t)}. \tag{26}$$

$\psi(t+1)>0.5$ always holds because $\psi(t)>0.5$ by assumption, showing no leapfrogging in this regime. Then, system (26) has a unique steady state equal to $1/(1+\mu)$. The condition $K^A(t)/K^B(t)>\Theta$ (Lemma 2) is equivalent to $\psi(t)>\Theta/(1+\Theta)$. Therefore, if $1/(1+\mu)>\Theta/(1+\Theta)$, then the steady state exists in the full North–South regime, which can be easily shown as stable through a usual phase diagram analysis. Similarly, we can show that if $1/(1+\mu)<\Theta/(1+\Theta)$, the steady state does not exist within the regime; $\psi(t)$ decreases over time and eventually becomes below Θ. Note that $1/(1+\mu)>\Theta/(1+\Theta)$ implies $\mu<1/\Theta$. ■

Next, we investigate the most important case (2), in which both countries innovate and only the lagging country produces goods. Since innovation takes places in both countries, the free-entry condition requires $V^A(t)=V^B(t)=0$ in equilibrium, which directly implies $w^A(t)/K^A(t)=w^B(t)/K^B(t)$ by (6). In this case, it holds that $N^A(t)>0$, $N^B(t)>0$, and $N(t+1)=N^A(t)+N^B(t)$. For convenience's sake, we refer to this specialization pattern as a North–North regime. Substituting the Euler equation (3) into the value of innovation (10), the free-entry condition $V^A(t)=V^B(t)=0$ implies

$$\frac{\beta\theta E(t)}{N(t+1)}=w^A(t)k^A(t)=w^B(t)k^B(t). \tag{27}$$

The interpretation of (27) is similar to that of (11). The second equality in (27) indicates that the cost of an innovation becomes internationally equated in equilibrium.

To determine the innovation flows in the present case,we apply $\Lambda^A(t)=\emptyset$ to the labor condition (12) in order to obtain

$$N^A(t)=LK^A(t). \tag{28}$$

By using (4) and monopolistic pricing $p^B(t)=w^B/\,(1-\theta)$, we obtain $x^B(t)=(1-\theta)E(t)/(N(t)w^B(t))$. Substituting this into the labor market condition (12) with (27), we can determine the innovation flow for lagging country B as

$$N^B(t)=\frac{1}{1+\Theta}\left(\Theta LK^B(t)-LK^A(t)\right). \tag{29}$$

Equation (29) shows that the innovation flow in the lagging country $N^B(t)$ increases with domestic knowledge stock $K^B(t)$ but decreases with foreign knowledge stock $K^A(t)$. To ensure that the lagging country also innovates in equilibrium, i.e., $N^B(t)>0$, by (29), it must hold that

$$\Theta>\frac{K^A(t)}{K^B(t)}. \tag{30}$$

This condition means that the international technological gap $K^A(t)/K^B(t)$ is not very large. Since, by assumption, $K^A(t)>K^B(t)$, (30) implies

$$\Theta>1. \tag{31}$$

Lemma 3 *The economy falls in the North–North regime if and only if* $\Theta > K^A(t)/K^B(t) > 1$, *in which both countries innovate and only the lagging country produces goods.*

Proof. See Appendix A. ■

Lemma 3 shows that if and only if leading country A's technological advantage, $K^A(t)/K^B(t)$, is not excessively large (condition (30)), then lagging country B can innovate in equilibrium by using the wage differential, which, as described above, exists in equilibrium so long as the fundamental profitability Θ of innovation is sufficiently large (condition (31)).

Using a numerical exercise, we examine how likely it is that the parameter condition (31) holds. From (15), the fundamental profitability of innovation is more likely to exceed 1 if consumers' time preference rate β is higher and/or the markup ratio for monopoly firms $1/(1-\theta)$ is higher. As already discussed, the length of one period in our model is fairly long (around 20 years), meaning that the time preference rate would be significantly lower than a standard value such as 0.98. Roughly, let us consider $\beta = 0.7 (\simeq (0.98)^{20})$, in which case the condition becomes $1/(1-\theta) > 2.427$. This is higher than the industry average but is included in the estimates by Hall (1986), who finds that in many industries, the markup ratio is above 1.5, and in some industries in the US between 1949 and 1978, it was higher than 2.5. One may think that although the condition $\Theta > 1$ holds for a moderate range of parameter values, it is not very likely.

Along with Lemmata 1–3, we demonstrate when and how the economy falls within a particular regime, which could be compared to reality as follows. Countries in the North–North regime could be related to developed countries, given the reality of higher R&D expenditure in developed countries. Countries in the (full) North–South regime could also be seen as developed and developing countries, given the standard consideration that a developed country has a higher income than a developing country. Note that in our model, the leading North's national income is typically higher than the lagging South's.[29]

Finally, we prove that leapfrogging may take place in equilibrium. To achieve this, we suppose that country A retains leadership for two consecutive periods in the North-North regime. That is, $\Theta > K^A(s)/K^B(s) > 1$ holds for two periods, $s = t$ and $t-1$. This implies that spillovers $M^B(t)$ are equal to $N^A(t-1) = LK^A(t-1)$ because innovations developed by country A in period $t-1$ all flow to lagging country B.

By substituting (28) and (29) into (9), with $M^A(t) = 0$ and $M^B(t) = LK^A(t-1)$, the growth of knowledge follows

$$K^A(t+1) = \underbrace{LK^A(t)}_{N^A(t):\text{ domestic innovation}} + K^A(t), \tag{32}$$

$$K^B(t+1) = \underbrace{\frac{\Theta LK^B(t) - LK^A(t)}{1+\Theta}}_{N^B(t):\text{ domestic innovation}} + \underbrace{\frac{\mu L}{L+1}K^A(t)}_{M^B(t):\text{ spillovers}} + K^B(t). \tag{33}$$

In (33), lagging country B has two sources of knowledge growth, namely, domestic innovation $N^B(t)$ and spillovers from foreign innovation $M^B(t)$, which sharply contrast with the North–

[29] See Appendix B (not for publication) for the formal proof. In Appendix B, we also derive growth rates in the three regimes, which are all different in general. Specifically, the growth rate in the North–North regime can be higher or lower than the rates in other regimes, depending on the parameters and international technology ratio $\psi(t)$.

South regime, wherein the lagging country does not innovate. By combining (32) and (33), we derive the international dynamics of knowledge as

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{\frac{\mu L}{L+1}\psi(t) + \left(\frac{\Theta L}{1+\Theta} + 1\right)}, \tag{34}$$

given $0.5 < \psi(s) < \frac{\Theta}{1+\Theta}$ for $s = t,\, t-1$, which is equivalent to $1 < K^A(t)/K^B(t) < \Theta$.

Using (34), the following theorem formally proves the perpetual cycle of leapfrogging as an equilibrium phenomenon.

Theorem 3 (Leapfrogging cycles with higher profitability) *Suppose that the fundamental profitability of an innovation Θ exceeds 1 and that the international technological gap $K^A(s)/K^B(s)$ is lower than Θ for $s = t,\, t-1$. Then, under dynamic optimization by the infinitely lived agent, both the leading and lagging countries innovate in equilibrium (North–North regime). In this case, neither country may successfully retain its technological leadership for infinite sequential periods; i.e., leapfrogging may take place repeatedly and perpetually along an equilibrium path. Specifically, this occurs if and only if*

$$\mu \in \left(\frac{2\,(L+1)}{1+\Theta}, 1\right]. \tag{35}$$

Proof. First, by Lemma 3, $\Theta > K^A(s)/K^B(s) > 1$ ensures that both countries innovate but only the lagging country produces goods. The steady state of system (34) is uniquely given by

$$\psi^* = \frac{1}{\mu}\frac{L+1}{1+\Theta},$$

which is stable. The steady state $\psi^*$is less than 0.5 if and only if (35) holds. Then, if and only if $\psi^* < 0.5$, given (34), $\psi(t)$ will stably decrease and eventually fall below 0.5, where country A loses its technological leadership. This means that when country A has leadership for two periods (t and $t-1$), it can never retain its leadership for an infinite number of sequential periods if and only if (35) holds. In other words, technological leadership is always temporary. By symmetry, it is straightforward to show the opposite case where country B initially has leadership for two periods. This proves the perpetual occurrence of leapfrogging, taking into account the fact that when a country initially has leadership for only one period, it either retains leadership for two periods or is immediately leapfrogged. ■

By combining Theorems 1–3, we reach our main result as follows:

Corollary 1 *If and only if (35) holds, the economy will eventually experience leapfrogging and its cycles along an equilibrium path for any initial condition on $(K^A(t), K^B(t))$.*

Proof. For sufficiency: If (35) holds, $\mu > 1/\Theta$ and $\Theta > 1$ must hold. Since $\Theta > 1$, by Lemmata 2 and 3, the economy falls within the full North–South regime or the North–North regime. In the case where the economy is in the full North–South regime (i.e., $K^A(t)/K^B(t) > \Theta$), since $\mu > 1/\Theta$, a path of $K^A(t)/K^B(t)$ eventually becomes below Θ, entering the North–North regime (Theorem 2). In the case where the economy is in the North–North regime (i.e., $\Theta > K^A(t)/K^B(t)$), since (35) holds, leapfrogging eventually occurs (Theorem 3). For necessity: If (35) does not hold, leapfrogging does not occur, regardless of whether $\Theta > 1$ (see Theorems 2 and 3) or $\Theta < 1$ (see Theorem 1). ■

One notable feature in the main result is that (35) is the necessary and sufficient condition for leapfrogging to occur in equilibrium. Therefore, it would be fruitful to discuss the economic implications of this condition in further detail. Condition (35) is more likely to hold if the spillovers μ are more efficient, if the country size L is smaller, and/or if the fundamental profitability of innovation Θ is larger. It is straightforward that the possibility of leapfrogging is increased by a rise in the efficiency of spillovers μ. More importantly, even if the lagging country can learn from the leading country's innovations with the same efficiency with which the leading country itself learns (i.e., $\mu = 1$), leapfrogging may not always occur. The key parameters are L and Θ, which relate to the two engines of knowledge accumulation, as explained below: country size L relates to "learning from foreign innovations" and fundamental profitability Θ relates to "domestic innovations."

To explain the role of country size L, we rewrite the spillovers the lagging country receives as

$$M^B(t) = N^A(t-1) = LK^A(t-1) = L\underbrace{\frac{K^A(t)}{L+1}}_{K^A(t-1)},$$

where the last equality comes from (28).[30] The essential element here is that the lagging country learns from the *past* innovations made by leading country A, $N^A(t-1)$, which are manufactured in the present period t. The term $\frac{1}{L+1}$ captures the fact that the leading country's knowledge grows at the rate of $L+1$ through $K^A(t) = (L+1)K^A(t-1)$. As the country size L is large, the leader's innovations grow more sharply from $t-1$ to t; the intertemporal gap $K^A(t) - K^A(t-1)$ is also larger. This intertemporal gap creates an intercountry gap of knowledge accumulation: while the leading country can learn from its *current* innovation $N^A(t) = LK^A(t)$, the lagging country can only learn from the leader's *past* innovation $N^A(t-1) = \frac{LK^A(t)}{L+1}$, which is depreciated by the growth rate $L+1$. When the country size L is large, this depreciation is also large, reducing the chance of leapfrogging.

Next, we explain the role of fundamental profitability Θ. Regardless of the efficiency of the spillovers μ, leapfrogging cannot take place if $\Theta < 1$. This limit reflects the fact that the lagging country does not innovate for itself if $\Theta < 1$. That is, the spillovers from foreign innovations alone can never bring about leapfrogging. Coexistence of domestic innovation and learning from foreign innovation is essential in the lagging country, which is the case in equilibrium if $\Theta > 1$. However, even in this case, leapfrogging does not occur when Θ is not sufficiently large to satisfy (35). When Θ becomes sufficiently large, the lagging country responds by developing more innovations (see (33)), while the leading country does not change its equilibrium behavior because it has already developed innovations at a maximum (see (32)). For a sufficiently large Θ, leapfrogging becomes inevitable. Note that $\Theta \equiv \beta\theta/(1-\theta)$ is increasing in the time preference rate β and the markup $1/(1-\theta)$. That is, when consumers are more patient, in that they prefer saving for future consumption (i.e., investment in innovation) to current consumption, and/or the monopolist can earn more

[30] Country A is assumed to be a leading country in period $t-1$. Since we focus on an identical equilibrium path, $K^A(t-1)$ must satisfy $K^A(t) = (L+1)\,K^A(t-1)$ by (28), given $K^A(t)$.

from an innovation owing to a higher markup, the lagging country innovates more actively in equilibrium, resulting in a higher possibility of leapfrogging.

Thus far, we have characterized the cyclical occurrence of leapfrogging with four parameters, μ, β, L, and θ. Finally, we briefly discuss how these relate to policy. Subjective time preference β and country size L should be seen as core parameters in the present context; from the descriptive perspective, leapfrogging cycles exist in equilibrium if and only if consumers are sufficiently patient and the country size is sufficiently small for the above-mentioned reason. The spillover efficiency μ is naturally related to the so-called absorptive capability (Cohen and Levinthal 1989) of firms in the host country to FDI, consistent with the empirical evidence that spillovers through FDI increase with the host country firms' degree of absorptive capability (see, for example, Borenszteina, Gregorio, and Lee 1998). In the empirical literature, the absorptive capability of a host country is often measured by its stock of human capital, which crucially depends on a country's educational system. In addition, absorptive ability depends on literacy and foreign language skills (Rogers 2004). Thus, we may relate the spillover parameter μ to general and foreign language education policy; the government would be able to raise μ by investing more in education, for instance, through a lump-sum tax, which increases the possibility of leapfrogging in our model. In a different context, we may relate the markup ratio $1/(1-\theta)$ to patent protection. Following the optimal patent policy literature (Gilbert and Shapiro 1990), we focus on a patent's breadth, interpreted as the patentee's ability to raise price, as a measure of patent protection. To formally model patent breadth, we need another parameter denoting an upper bound on the markup; see, for example, Li (2001), Goh and Oliver (2002), Chu (2011), and Iwaisako and Futagami (2013) for research in this line. With this limitation in mind, we consider that a higher markup ratio $1/(1-\theta)$ captures a larger patent breadth. Hence, a tightening of patent protection in the sense of a larger patent breadth increases firms' profit, which encourages the lagging country to innovate more actively, raising the possibility of leapfrogging.

We summarize the implication of our result as a proposition.

Proposition 1 *An economy will eventually experience leapfrogging and its cycles over time if and only if (a) the consumers are patient and prefer saving to current consumption, (b) the country's size is small, (c) the efficiency of international knowledge spillovers is reasonably high, and (d) the markup ratio is high. Condition (c) could be related to a larger absorptive capability of firms, while condition (d) could be related to stronger patent protection.*

The key driving force behind leapfrogging cycles is a lagging country's dual growth engine. In the North–North regime, the lagging country both innovates and manufactures. Thus, the lagging country's knowledge accumulates not only through its own innovations but also through the flow of spillovers from the leading country's innovations. In this sense, the growth engine of knowledge in the lagging country is twofold: innovating autonomously and learning from abroad. Although the leading country innovates faster than the lagging country, knowledge growth in the leading country is driven only by domestic innovations, which creates the possibility of leapfrogging. This mechanism of leapfrogging works under (35) and is enhanced by the four factors, (a)–(d), in Proposition 1.

3.4 An illustration

To illustrate, we again use a phase diagram. However, the configuration of the phase diagram depends on the history, i.e., which country was a leading country in the previous period. As this is simply a problem of visual complication, to clarify the illustration, we assume that innovation activities are completed within one period. Thus, the innovation value in (6) should be replaced by

$$V^i(t) = \max\{\pi^A(t), \pi^B(t)\} - w^i(t)k^i(t). \tag{36}$$

In this modified setting, the necessary and sufficient condition for the North-North (full North-South) regime given in Lemma 3 (Lemma 2) is simplified to $\theta > \psi(t) > 0.5$ ($\psi(t) > \theta > 0.5$). Recall that θ is the parameter that determines the elasticity of substitution. Then, we can describe the international dynamics of knowledge as follows.[31]

$$\psi(t+1) = \Phi(\psi(t)) \equiv \begin{cases} \frac{\mu L + (1-\mu L)\psi(t)}{1+(1+\mu)L(1-\psi(t))} & \text{for } \psi(t) \in (0, 1-\theta) \\ \frac{(\theta-(1-\mu))L+(1+(1-\mu)L)\psi(t)}{\theta L + 1 + \mu L(1-\psi(t))} & \text{for } \psi(t) \in (1-\theta, 0.5) \\ (L+1)\frac{\psi(t)}{1+\theta L + \mu L\psi(t)} & \text{for } \psi(t) \in (0.5, \theta) \\ (L+1)\frac{\psi(t)}{1+(1+\mu)L\psi(t)} & \text{for } \psi(t) \in (\theta, 1) \end{cases}. \tag{37}$$

The equilibrium dynamic system Φ is autonomous and nonlinear. Figure 2 depicts the phase diagram of system Φ for $\mu < 2(1-\theta)$. There are two steady states, both of which are stable. For all initial points, technological leadership can never alternate internationally. In this case of $\mu < 2(1-\theta)$ where the spillovers are less efficient (small μ), the result is essentially identical to that in the North–South regime; that is, no leapfrogging takes place.

There are two subcases with (a) $\mu < (1-\theta)/\theta$ and (b) $(1-\theta)/\theta < \mu$. In case (a), even if the advantage of the leading country is initially very small ($\psi(t)$ is around 0.5), the knowledge gap steadily widens and the world economy finally converges to the steady state (ψ_i^{**}) in the full North–South regime. The two countries, even though both innovate initially, will eventually split into innovative and non-innovative countries, in which the outcome is ultimately determined by the initial (small) knowledge difference. In case (b), $\psi(t)$ converges to the steady state (ψ_i^*), in which case both countries innovate in the long run.

Most importantly, Figure 3 depicts a typical path for the case in which $\mu > 2(1-\theta)$. Given that no steady state exists, the international knowledge fraction $\psi(t)$ will perpetually move back and forth between the two regimes $(0, 0.5)$ and $(0.5, 1)$. Finally, note that the condition of leapfrogging cycles in the simplified model, $\mu > 2(1-\theta)$, is analogous to (35).

4 Discussion

As previously mentioned, albeit tangentially, our paper has several limitations that arise from the model's specification. First, specialization takes place in the present model, which is a dynamic version of the Ricardian model. In reality, the leading country also manufactures foreign innovations (those in the lagging country) and may also learn from them. In addition, historically, no country has specialized in R&D. Therefore, this model captures only

[31] See Appendix A for the derivations.

a certain aspect of real-world behavior. That is, lagging countries may have an advantage in international technological competition with the leading country because they can learn from the leader's active innovation as well as their own innovation experience. We can easily remove this unrealistic aspect concerning specialization from the model by assuming, for instance, a strictly concave production function in manufacturing. As this would make the analysis intractable without adding new insights, we adopt the present setting for simplicity.

Second, given the historical fact that technological leadership has often shifted between countries, it is important to provide an extended case comprising more than two countries. We can demonstrate that three or more countries on an equilibrium path can perpetually experience various forms of leapfrogging, including, for example, growth miracles (Matsuyama 2007), in which the least productive country leapfrogs all rival countries having higher productivity levels in a single burst. Such growth miracles may take place sporadically or consecutively or in some complex combination; see Furukawa (2012) for a formal analysis.

Third, to clearly explain the mechanism through which leapfrogging occurs along an equilibrium path, we assume that countries are basically homogeneous. Allowing for country heterogeneity, we can demonstrate that leapfrogging may take place finite times in the model where countries have different labor endowments and/or efficiency levels of spillovers.[32]

Fourth, because the model assumes discrete time, it is implicitly assumed that the leading country takes a long time to exploit its technological advantage, a delay that allows the lagging country to leapfrog it. One may wonder *why the leading country waits so long.* The simple answer is that, in each period, firms in the leading country only pursue a one-period monopoly; thus, they do not worry about whether the leadership of their home country persists. For example, if we introduce into the model a welfare-maximizing government that gives subsidies to innovating firms, a policy game between international governments may lead to a different situation where the leading country does not wait so long. However, because innovations often take a very long time from startup to implementation and emerge in bunches,[33] asymmetric information prevents the government in the leading country from seeing *in real time* what is happening in the lagging country. Given this, the leading country can only wait and see what happens in the lagging country.

Of course, it is potentially necessary to extend our discrete-time analysis to continuous time. In a continuous-time setting, it is essential to consider what happens as the technology levels of two countries become equal in the process of leapfrogging. One way forward would be to focus on technological complementarity between countries. Spillovers from the leading country then combine with the backward technology of the lagging country, meaning that our leapfrogging mechanism should work in a continuous-time setting. We leave this fundamental issue associated with discrete time for future research.

Fifth, we should revisit the lack of factors such as stochastic innovation, costly adoption of innovation, and strategic interactions, which should be a limiting factor in a broader context. As we previously mentioned, if we implement a very simple setting, we can consider a stochastic innovation process or a costly adoption process without essentially changing the results.[34] Meanwhile, although an introduction of stochastic *timing* of innovation or strategic

[32] The formal analysis is available from the author upon request.

[33] This issue is intensively investigated in the literature on innovation cycles (see Shleifer 1986).

[34] See Appendix C (not for publication).

interactions between firms could be interesting, it makes the analysis intractable.

Sixth, we consider FDI flows as the only spillover channel. In the model, FDI is allowed to flow in both directions but takes place only from the leader to the laggard in equilibrium; as a result, knowledge flows in that direction only. One might regard this equilibrium characteristic of one-way knowledge spillovers as unrealistic, given that the two countries are identical except for their knowledge stock for innovation. However, if we allowed a more general spillover function, including laggard-to-leader knowledge spillovers, our main mechanism would not cease and the results would still hold under original condition (35) with one extra condition. Specifically, we can extend the model by adding spillovers that are *not* based on FDI to the present setting. In such an extended model, knowledge flows in both directions *in equilibrium*; under (35), we can demonstrate that leapfrogging occurs if the efficiency of non-FDI spillovers is not very large.[35] Therefore, we can say that our simple specification on spillovers in (8) captures a general aspect of the leapfrogging phenomenon.

Finally, the mechanism of leapfrogging cycles shown here is endogenous in that it depends on an endogenous process of knowledge accumulation, which is based on Romer's (1990) setting. However, it should be noted that it is not endogenous in a strict sense since the knowledge accumulation process we consider does not explicitly incorporate any economic incentive or profit motive but rather assumes externalities of past innovations on current technological productivity, so that the learning *mechanism* itself is essentially exogenous. In a complete model of fully endogenous leapfrogging cycles, we would consider voluntary investment activity by firms to learn from foreign innovations; we leave this for future research.

5 Concluding Remarks

In this paper, we developed a two-region endogenous innovation model with dynamic optimization of the infinitely lived consumer, in which knowledge diffuses internationally through FDI. The major finding is that technological leadership may shift internationally, perpetually moving back and forth between countries if the profitability of an innovation is higher and the spillovers are relatively efficient. Specifically, if the profitability of an innovation is lower, in equilibrium, only the leading country innovates. In this case, leapfrogging never arises. If the profitability of an innovation is higher, in equilibrium, both countries innovate. In this case, leapfrogging perpetually takes place along an equilibrium path if international spillovers are reasonably efficient. In a big picture, we may say that the growth process of an international economy can be intrinsically cyclical depending on the factors such as the profitability of innovation and the efficiency of international spillovers.

Our result shows the possibility that lagging countries leapfrog *in spite of innovating less*, by focusing on learning from foreign innovation as a missing link. In this sense, the present paper is close to Glass (1999) in spirit. With some examples from Asian countries including South Korea or China,[36] Glass (1999) considers whether learning from foreign innovation through imitation can serve as a stepping stone enabling firms from lagging countries to undertake innovation. The present paper extends Glass's (1999) view by demonstrating that

[35] We show this in Appendix C (not for publication).

[36] See Carolan, Singh, and Talati (1998).

the effectiveness of learning may be a key factor in enabling the lagging country to leapfrog the leading country by shifting to a more innovative economy.

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Appendix A

Proof of Lemmata 1–3. The necessity has been shown in the text. Namely, $\Theta < 1$ must hold if both countries produce, recalling that only the leading country innovates in equilibrium if both countries manufacture. Here, we prove sufficiency, i.e., both countries produce in equilibrium if $\Theta < 1$. For this purpose, let us prove the contrapositive: we show $\Theta > 1$ must hold if only one country manufactures. There are two cases. (a) If only leading country A manufactures in equilibrium, $w_A(t) < w_B(t)$ must hold. With $K^A(t) > K^B(t)$, this implies $w^A(t)/K^A(t) < w^B(t)/K^B(t)$, under which only leading country A innovates owing to its lower R&D cost. There is no labor demand in country B, so (a) cannot be an equilibrium. (b) If only lagging country B manufactures in equilibrium, $w_A(t) > w_B(t)$ must hold. Case (b) has three sub-cases. (b-i) Only lagging country B innovates, which is inconsistent for the same reason as in (a). (b-ii) If only leading country A innovates in equilibrium, $w^A(t)/K^A(t) < w^B(t)/K^B(t)$ must hold. The labor market condition (12) implies $N^A(t) = LK^A(t)$ and $w^B(t) = (1-\theta)\,E(t)/L$.[37] The free-entry condition requires $V^A(t) = 0$ because innovation takes place in equilibrium for country A, which implies $w^A(t) = \beta\theta K^A(t)E(t)/N(t+1)$.[38] Noting $N(t+1) = N^A(t)$ in this case, $w^A(t) > w^B(t)$ implies $\Theta > 1$, not $\Theta < 1$. (b-iii) If both countries innovate in equilibrium, it must be by the free entry condition that $V^A(t) = V^B(t) = 0$ in equilibrium, which implies $w^A(t)/K^A(t) = w^B(t)/K^B(t) = \frac{\beta\theta E(t)}{N(t+1)}$.[39] Using this and $x^B(t) = \frac{(1-\theta)E(t)}{N(t)w^B(t)}$ from (4), $N^B(t) = \left(\Theta K^B(t) - K^A(t)\right) L/(1+\Theta)$ is derived from (12). Since $N^B(t) > 0$, then $\Theta > 1$ must hold, noting $K^A(t) > K^B(t)$. We have proven that if $\Theta < 1$, both countries must manufacture in equilibrium; thus, only the leader innovates. We can easily prove Lemmata 2 and 3 in an analogous way (see Appendix B (not for publication) for a formal proof). ■

Derivations for (37)

In the North–North regime: there are two cases (A) and (B). (A) Assume $\frac{\theta}{1-\theta} > \frac{K^A(t)}{K^B(t)} > 1$ (i.e., $\psi(t) \in (0.5, \theta)$), where country A is a leading country. The free-entry condition (27) becomes

$$\frac{\theta E(t)}{N^A(t) + N^B(t)} = \frac{w^i(t)}{K^i(t)}. \tag{A1}$$

Given the labor condition for leading country A, we have

$$N^A(t) = LK^A(t). \tag{A2}$$

By the labor condition for the lagging country, $L = \left(N^B(t)/K^B(t)\right) + \left(N^A(t) + N^B(t)\right) x^B(t)$, with $x^B(t) = \frac{(1-\theta)E(t)}{(N^A(t)+N^B(t))w^B(t)}$, we thus have

$$N^B(t) = \theta LK^B(t) - (1-\theta)\, LK^A(t), \tag{A3}$$

[37] By (2), we can derive $x^B(t) = (1-\theta)\,E(t)/(N(t)w^B(t))$. Substituting this into $L = N(t)x^B(t)$ yields this expression.

[38] By (5),$\pi^B(t) = \theta E(t+1)/N(t+1)$. Incorporating this and (3) into (6), $V^A(t) = 0$ implies this expression.

[39] Note (6), together with $\pi^B(t) = \frac{\theta E(t)}{N(t)}$ from (5), $\pi^B(t) > \pi^A(t)$, and (3).

in which $w^B(t)$ is eliminated using (A1). Noting $M^B(t) = N^A(t)$ here, by (9), we have the dynamic system as follows:

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{\mu L\psi(t) + \theta L + 1} \text{ for } \psi(t) \in (0.5, \theta)\,. \tag{A4}$$

Note that $\theta > 0.5$ holds in the North–North regime. (B) Assume $\frac{1-\theta}{\theta} < \frac{K^A(t)}{K^B(t)} < 1$ (i.e., $\psi(t) \in (1-\theta, 0.5)$), where country B is a leading country. Due to the symmetry,

$$N^A(t) = \theta LK^A(t) - (1-\theta)\,LK^B(t) \text{ and } N^B(t) = LK^B(t) \tag{A5}$$

hold. We can also derive

$$\psi(t+1) = \frac{(\theta + (\mu - 1))L + (1 + (1-\mu)\,L)\,\psi(t)}{\theta L + 1 + \mu L\,(1-\psi(t))} \text{ for } \psi(t) \in (1-\theta, 0.5)\,. \tag{A6}$$

In the full North–South regime, there are two cases (C) and (D). (C) Assume $\frac{K^A(t)}{K^B(t)} > \frac{\theta}{1-\theta} > 1$ (i.e., $\psi(t) \in (\theta, 1)$). The leading country innovates following

$$N^A(t) = LK^A(t) \text{ and } N^B(t) = 0. \tag{A7}$$

The lagging country receives spillovers $\mu M^B(t) = \mu N^A(t)$, with $\mu \leq 1$.[40] Then, the knowledge dynamics are as follows:

$$\psi(t+1) = \frac{(L+1)\,\psi(t)}{1 + (1+\mu)\,L\psi(t)} \text{ for } \psi(t) \in (\theta, 1)\,. \tag{A8}$$

(D) Assume $\frac{K^A(t)}{K^B(t)} < \frac{1-\theta}{\theta} < 1$ (i.e., $\psi(t) \in (0, 1-\theta)$). Due to the symmetry, $N^A(t) = 0$ and $N^B(t) = LK^B(t)$, with $\mu M^A(t) = \mu N^B(t)$. We can easily have

$$\psi(t+1) = \frac{\mu L + (1-\mu L)\,\psi(t)}{1 + (1+\mu)\,L\,(1-\psi(t))} \text{ for } \psi(t) \in (0, 1-\theta)\,. \tag{A9}$$

[40] By using the labor market condition for lagging country B and the free-entry condition, we can easily verify that

$$\frac{A^A(t)}{A^B(t)} > \frac{w^A(t)}{w^B(t)} = \frac{\theta}{1-\theta} > 1$$

holds.

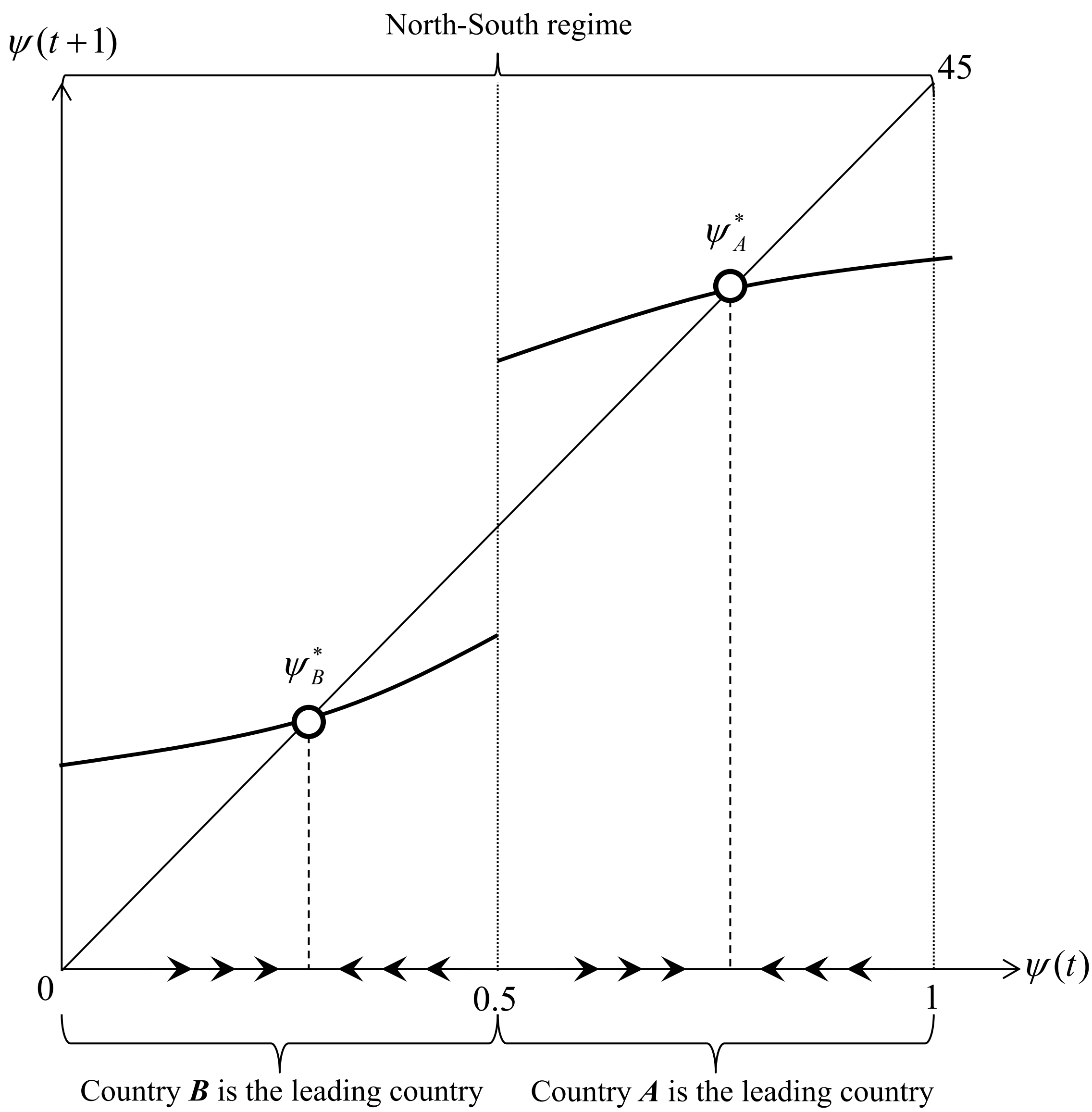

Figure 1: No leapfrogging in the North-South regime

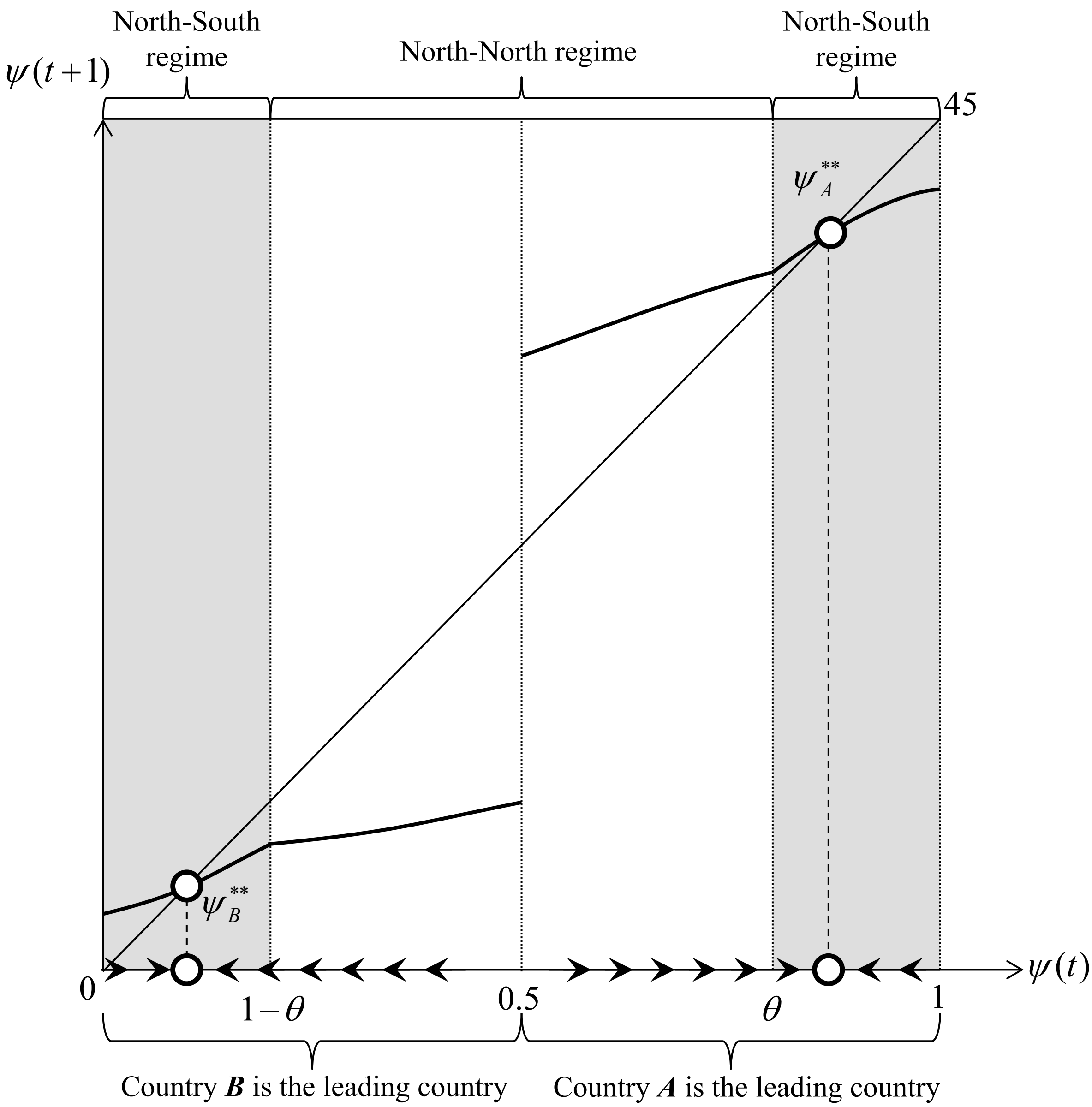

Figure 2: No leapfrogging in the North-South regime
(a) Converging to ψ_i^{**} as $\mu < (1-\theta)/\theta$

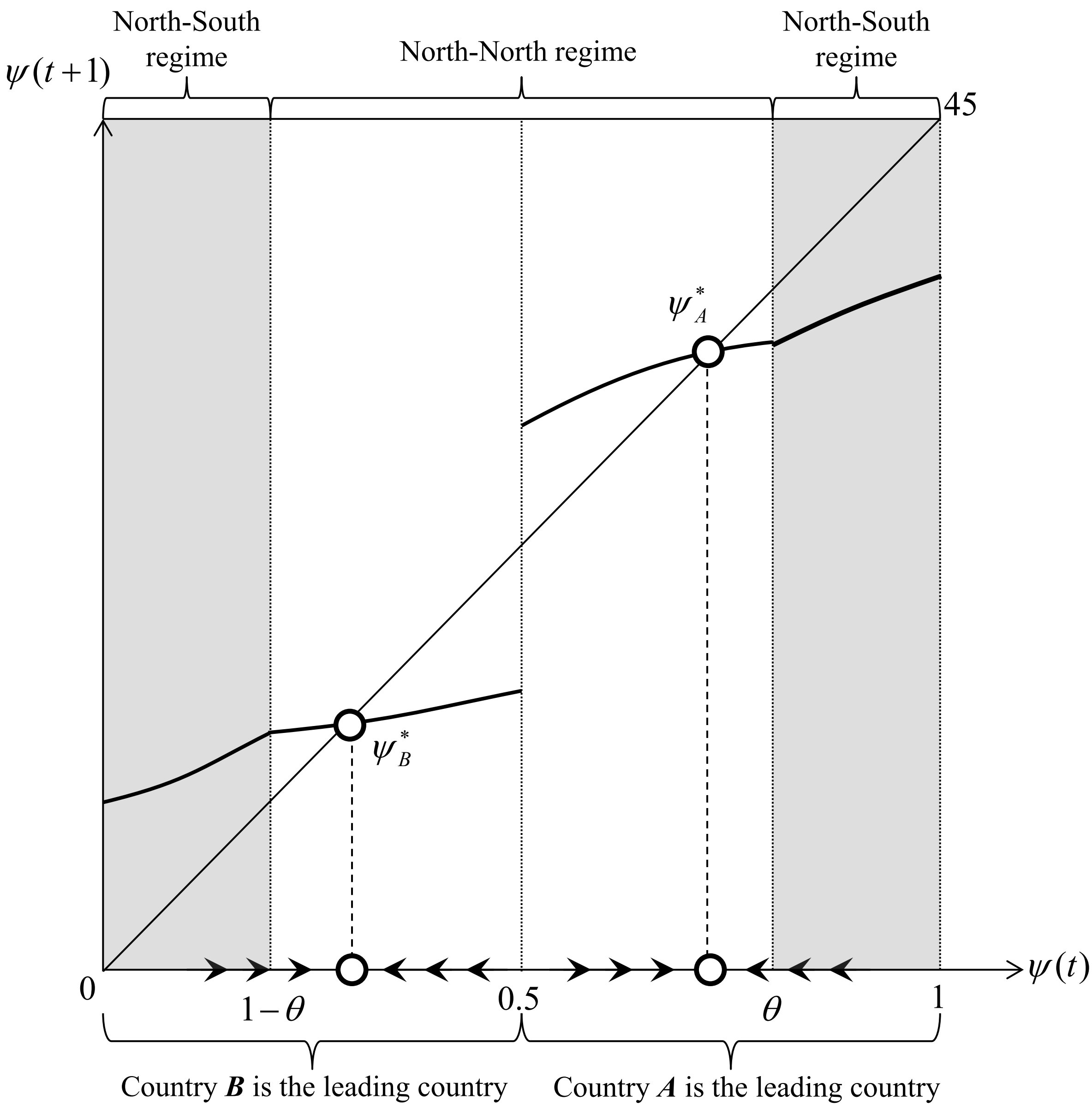

Figure 2: No leapfrogging in the North-North regime
(b) Converging to ψ_i^* as $\mu > (1-\theta)/\theta$

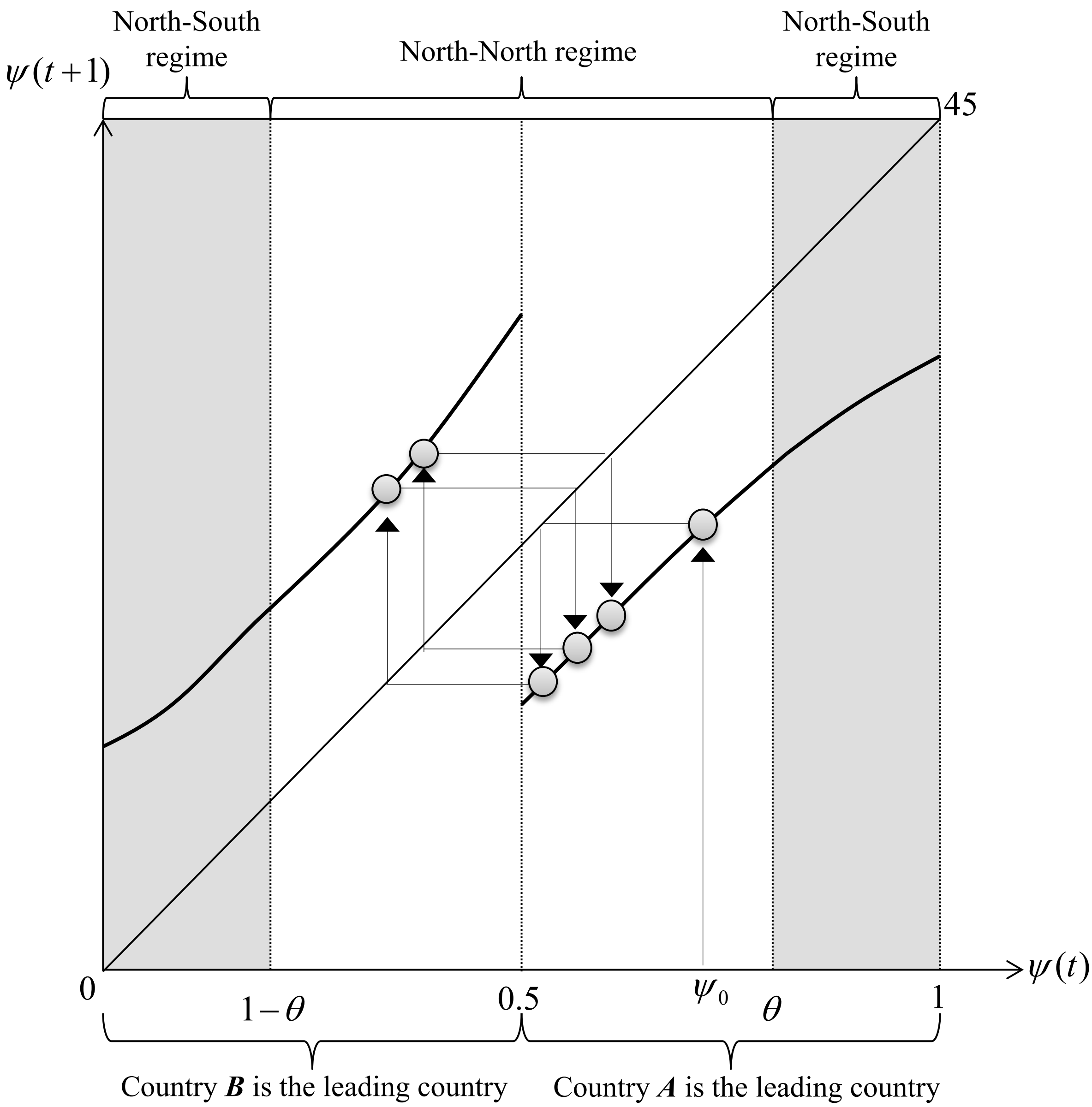

Figure 3: Leapfrogging cycles in the North-North regime